

MICROWAVE BACKGROUND ANISOTROPIES, LARGE-SCALE STRUCTURE AND COSMOLOGICAL PARAMETERS

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Abstract

We review how the various large-scale data constrain cosmological parameters and, consequently, theories for the origin of large-scale structure in the Universe. We discuss the form of the power spectrum implied by the correlation data of galaxies and argue by comparing the velocity field implied by the distribution of light with the observed velocity flows that the bias parameter, b , is likely to be constant in the linear regime. This then allows one to estimate the density parameter, Ω , and b directly from the data on $\xi(r)$ and the velocity fields. We show that it is consistent with low values of $\Omega^{0.6}/b$. We discuss the ways to normalise the optical data at $z \sim 0$ directly to the COBE (or other microwave background) data. The data on high- z galaxies allows one to further constrain the shape of the primordial power spectrum at scales which are non-linear today ($< 8h^{-1}\text{Mpc}$) and we discuss the consistency of the data with inflationary models normalised to the large-scale structure observations.

1 Introduction

The purpose of this review is to discuss the constraints the current astronomical data place on the values of cosmological parameters, such as Ω , the primordial power spectrum $P(k)$ etc. Indeed, the last few years brought wealth of observational data in optical, radio and microwave bands that allow now to constrain these parameters more tightly and thus to gain further insight into the early Universe physics, particularly in light of the inflationary picture. We first discuss the implications for the inflationary scenario of the (realistic) possibility that the Universe may turn out to be open. We point out that the Grischuk-Zeldovich (1978) effect combined with the COBE observations of the quadrupole anisotropy of the microwave background radiation (MBR) would then preclude the possibility that the Universe’s homogeneity

was produced by inflationary expansion during its early evolution. Next, in Section 3 we discuss the constraints on the primordial power spectrum of the light distribution on scales $10 - 100h^{-1}\text{Mpc}$ from the optical data at $z \sim 0$. Section 4 deals with comparing the (mass) power spectrum deduced from the peculiar velocities data and that of the light distribution. We point out that the two are consistent with each other and therefore it is indeed plausible to assume that they are proportional to each other, i.e. the bias factor b is constant. We further note that the comparison of the two leads to low values of $\Omega^{0.6}/b$. In Section 5 we discuss the normalisation of the power spectrum to the COBE results and whether the latter necessarily imply flat Universe and/or Harrison-Zeldovich spectrum of the primordial density fluctuations. Section 6 discusses the constraints on $P(k)$ on small scales, which are non-linear today, from the data on high redshifts galaxies and the Uson et al (1992) object. We conclude in Section 7.

2 Ω and inflation

The value of the density parameter, or more precisely the curvature radius of the Universe, is certainly the most critical test of inflation. Some data seems consistent with the flat Universe (Kellerman 1993 and these proceedings; Yahil 1993, these proceedings). But I think it is fair to say that most observational data seems to point to low values of Ω . In particular the age (t_0) measurements indicate that $H_0 t_0 > \frac{2}{3}$ (e.g. Lee 1992 and these proceedings) and dynamics of the Local Group strongly prefers $\Omega \sim 0.1 - 0.2$ (Peebles 1989 and Tully 1993, these proceedings; see also Kashlinsky 1992a and discussion in Sec.4).

What would it mean if the data were to prove that the Universe is old and open? There were some suggestions in the past attempting to accomodate the open Universe with the finely-tuned inflation (e.g. Ellis 1988; Steinhardt 1990), the idea being that the Universe has undergone only the minimal numbers of e -foldings (of order ~ 65) necessary to make it homogeneous on scales not much greater than the present horizon, $R_{hor} \sim 6000h^{-1}\text{Mpc}$. This would then make the curvature radius $\sim R_{hor}$ leading to Ω as low as $\sim 0.1 - 0.3$, but would also imply that we just happened to be born at the time when the horizon did not yet grow large enough to encompass more than the inflation-blown homogeneous bubble.

The problem with such finely-tuned inflationary models is that they would violate the COBE measurements of the quadrupole anisotropy as discussed by Frieman, Kashlinsky and Tkachev (1993) (see also Turner 1991). In that case the Universe would be inhomogeneous on scales greater than the horizon. If the amplitude of the superhorizon harmonics of lengthscale l is h , it would cause, via the Grischuk-Zeldovich (1978) effect, the quadrupole microwave background anisotropy of the amplitude $Q \simeq h(R_{hor}/l)^2$. Now both l and Ω are, in inflationary scenarios, functions of the number of e -foldings, N , while the amplitude of Q produced by the superhorizon inhomogenieties cannot exceed the COBE found value of $Q_{COBE} \simeq 5 \times 10^{-6}$. Thus one can express l in terms of Ω and then rewrite the constraint imposed by the Grischuk-Zeldovich effect and the COBE data directly as a constraint on the present value of the density parameter

(Frieman, Kashlinsky and Tkachev 1993):

$$1 - \Omega \leq \frac{Q_{COBE}}{h} \simeq O(10^{-6}) \quad (1)$$

The above means that if the Universe went through an inflationary phase during its evolution (so that $h \gg 1$ on scales where inflationary smoothing was inefficient), the COBE observations require it to have the density parameter within a factor $\sim 10^{-6}$ of unity. Conversely, if it turns out that observations prove that the Universe is open this would mean that the homogeneity of the Universe has not originated by the inflationary expansion.

3 Structure at $z \sim 0$ and the primordial power spectrum

The distribution of galaxies and galaxy systems (light) is now measured fairly accurately on scales up to $\sim 100h^{-1}\text{Mpc}$ from various and independent datasets. All the datasets give results which are in good agreement with each other and, if the light distribution is at least proportional to that of mass and all find substantially more (light) power on large scales than simple inflationary models (e.g. cold-dark-matter - CDM) would predict. As was mentioned, the data discussed in this section measure the distribution of light and in order to get information about the mass power spectrum one has to make assumptions about the interdependence between the light and mass distributions.

We discuss below in chronological order the most accurate datasets for determining the power spectrum of light on large scales:

1) Cluster-cluster correlation function was measured for Abell (Bahcall and Soneira 1983) and Zwicky (Postman et al 1986) clusters. The data showed that the correlation function of light remains positive on scales up to $\simeq 100h^{-1}\text{Mpc}$ and is roughly $\xi(r) \propto r^{-2}$ thus implying the power spectrum of light, $P(k) \propto k^n$, of $n \simeq -1$ (Kaiser 1984; Kashlinsky 1987,1991a). Furthermore, the data showed a systematic increase of the correlation amplitude with the cluster richness (or mass). This as suggested by Kashlinsky (1987,1991a) can be explained within the gravitational clustering model of cluster formation and the dependence of the increase on cluster masses requires $n \simeq -1$ and is inconsistent with the standard CDM model (Kashlinsky 1991a). There were some suggestions that the cluster-cluster correlation is strongly biased on large scales by projection effects (Sutherland 1988), but there is some evidence to the contrary (e.g. Szalay et al 1989). Furthermore, since the power implied by it is consistent with other and later findings discussed below, there is good chance it reflects a true distribution of light, if not of the mass itself.

2) The two-point galaxy correlation function of galaxies has been measured in the APM survey (Maddox et al 1991). The measurements were done in the b -band and covered approximately one square steradian of ~ 2.5 million galaxies. This is probably the most accurate measurement of the projected angular correlation function $w(\Theta)$ with the systematic error being less than 2×10^{-3} . The results show significantly more power than the standard CDM model would predict and imply $n < 0$ on scales $< 100h^{-1}\text{Mpc}$.

3) Picard (1991) has compiled the POSS catalogue in the r -band of approximately 400,000 galaxies in projection and determined $w(\Theta)$ which is roughly consistent with the APM survey. Similarly, the COSMOS machine results give $w(\Theta)$ in good agreement with the APM data and the cluster-cluster correlations (Collins et al 1992).

4) Redshift surveys allow one to map the galaxy distribution in the 3-D space, but here one has to correct for the distortions induced by the peculiar velocity flows (Kaiser 1988). Vogeley et al (1992) have measured the power spectrum of the light distribution from the CfA redshift survey and find results consistent with the ones mentioned above. Fisher et al (1993) find similar results from the catalogue of $\sim 5,000$ IRAS galaxies.

5) QDOT analysis of counts in cells (Saunders et al 1991) for $\sim 2,000$ IRAS galaxies gives results consistent with the above.

Thus the consistency of the above independent studies of different objects, in different wavebands and at different depths indicates a good probability of the reality of these results. Consequently it may make little sense to discuss cosmological models (e.g. CDM) in the context of one set of measurements only (e.g. COBE). One has to compare theories with all observational data, i.e. to normalise them simultaneously to the large-scale data, that map the distribution of matter at the present epoch ($z \sim 0$) and MBR observations which map the mass distribution at the last scattering surface (in the absence of reheating $z \sim 1100$). We devote the rest of this review to discussing how such normalisation can be done (Juszkiewicz et al 1987; Gorski 1991; Kashlinsky, 1991b;1992a,b,c) and what information it carries.

So what is the power spectrum (of light) implied by the large-scale structure data? Below, we use mainly the APM data, but as discussed above the other datasets would give consistent results. For small angles Θ the two-point correlation function $w(\Theta)$ decreases with Θ as $\Theta^{-\gamma}$ with $\gamma \simeq 0.7$. This implies a power spectrum $P(k) \propto k^{\gamma-2}$ for sufficiently large k (small scales). On larger angular scales $w(\Theta)$ falls off rapidly and its signal becomes lost in the systematic noise ($\simeq 2 \times 10^{-3}$). The fall-off may imply the following things: 1) the power spectrum goes into the white-noise regime ($n = 0$) leading to $\xi(r) \simeq 0$; 2) $P(k)$ goes into the Harrison-Zeldovich regime, $n = 1$, where the correlation function is negative and falls off rapidly with r : $\xi(r) \propto -r^{-4}$; 3) the power spectrum has the power index n even greater than the Harrison-Zeldovich value of 1 in which case the correlation function decreases even more rapidly with r : $\xi(r) \propto -r^{n+3}$ and the signal gets lost in the noise. Thus the only conclusion one can make from the fall-off in $w(\Theta)$ at large Θ is that these scales correspond to the power index $n \geq 0$ at sufficiently small k . Various forms for the fit were proposed by Kashlinsky (1991a,b) and Peacock (1991). Below we will adopt the form from Kashlinsky (1992c) which is particularly simple to use for quick estimates and which gives essentially the same results as the former two:

$$P(k) = \frac{2\pi^2 \xi(r_8)}{k_0^3 \Phi_\xi(k_0 r_8; n)} \times \begin{cases} (\frac{k}{k_0})^n, & k < k_0 \\ (\frac{k}{k_0})^{\gamma-2}, & k > k_0 \end{cases} \quad (2)$$

The above has been normalised to the observed value, $\xi(r_8) = (r_8/r_*)^{-\gamma-1}$ at $r_8 = 8h^{-1}\text{Mpc}$ and $r_* = 5.5h^{-1}\text{Mpc}$ and $\Phi(y, n) \equiv \int_0^1 x^{2+n} j_0(xy) dx + \int_1^\infty x^\gamma j_0(xy) dx$. The value of the transition scale, k_0^{-1} , must be determined from matching eq.(2) to

the APM data on $w(\Theta)$. Note, that because of the rapid fall-off in $\xi(r)$ for $n \geq 0$, the value of k_0^{-1} is rather insensitive to the precise value of the free parameter n as long as it remains positive on large scales. Kashlinsky (1992c) has analysed the APM data in the narrow magnitude slices, each slice $\Delta m \simeq 0.5$ wide, thereby reducing effects due to evolution of galaxies lying at different depths and finds $k_0^{-1} = 50h^{-1}\text{Mpc}$ from matching (2) to the APM data. Peacock's (1991) form for $P(k)$ would give similar results when applied to the non-scaled APM data in the narrow magnitude bins.

4 Normalising to velocity field: does light trace mass?

How is the power spectrum of the light distribution discussed in the previous section related to the power spectrum of mass, which if one accepts the inflationary prejudices is uniquely specified by the early Universe physics? Obviously within the framework of inflationary models, the transition scale to the Harrison-Zeldovich regime is not a free parameter and is approximately equal to the horizon scale at the epoch of the matter-radiation equality. It is thus expected that for such models k_0^{-1} should be $\simeq 13(\Omega h)^{-1}h^{-1}\text{Mpc}$, so that the correlation function of mass for the standard CDM model ($\Omega = 1, h = 1/2$) has zero crossing at $\simeq 30h^{-1}\text{Mpc}$. This is considerably smaller than the value of k_0^{-1} indicated by the data.

The answer to the above question comes from comparing the power spectrum implied by the peculiar velocity data (which map the mass distribution) with that of light discussed in the previous section. This has been done by Kashlinsky (1992a), who proposed a method to relate the velocity field directly to the correlation function, thereby eliminating the power spectrum entirely from discussion. He then computed the peculiar velocity field implied directly by the APM data on $w(\Theta)$ (assuming that the latter is proportional to the mass power spectrum) and compared the results with the Great Attractor peculiar field. The comparison is plotted on Fig.2 of Kashlinsky (1992a), which shows that the peculiar velocity field (its amplitude and coherence length) due to the APM data is in good agreement with the Great Attractor data. This therefore suggests that it is indeed reasonable to assume that at least in the linear regime (scales $r > 8h^{-1}\text{Mpc}$) the bias factor, b , can (and must?) be assumed to be constant with scale.

Furthermore, using the methods developed in Kashlinsky (1992a) one can determine the density parameter Ω and the bias factor b by comparing the peculiar velocity data with the velocity field predicted by the APM. For simplicity, we reformulate this method in terms of the three-dimensional correlation function, $\xi(r)$. The "dot" peculiar velocity correlation function is defined as $\nu(r) = \langle \mathbf{v}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x} + \mathbf{r}) \rangle$, and, if the light and mass power spectra are proportional to each other, the power spectrum can be eliminated from discussion and one can relate $\nu(r)$ directly to $\xi(r)$ (see Kashlinsky 1992a for details). The relation between the two is given by the second order differential equation:

$$\nabla^2 \nu(r) = -\frac{\Omega^{1.2}}{b^2} H_0^2 \xi(r) \quad (3)$$

As discussed in the previous section, the APM data suggest that the zero crossing of $\xi(r)$ occurs on large scale, $\simeq 2.5k_0^{-1} \simeq 130h^{-1}\text{Mpc}$. Thus, as follows from eq.(2), on

scales, on which the velocity data is most reliably determined ($\leq 60 - 70h^{-1}\text{Mpc}$) $\xi(r)$ is to a good approximation given by $\xi(r) \simeq (r/r_*)^{-\gamma-1}$. Using this approximation for $\xi(r)$ (rooted in observations), one can solve eq.(3) analytically using as one boundary condition the fact that $\nu(0)$ must be finite. Defining $V_* \equiv H_0 r_* = 550\text{km/sec}$, the solution is given by:

$$\nu(r) = \nu(0) - \frac{\Omega^{1.2}}{b^2} \frac{V_*^2}{(1-\gamma)(2-\gamma)} \left(\frac{r}{r_*}\right)^{1-\gamma} \quad (4)$$

The observed value of the dot velocity correlation function at zero is given by e.g. pairwise velocities, cluster velocity dispersions etc and is thought to be $\sqrt{\nu(0)} \simeq 500 - 700\text{km/sec}$ (e.g. Peebles 1987 and references cited therein). The lower end of that range would be in better agreement with the dipole MBR anisotropy (local) motion of $\simeq 630\text{km/sec}$. The data on $\nu(r)$ were determined on scales $\leq 60h^{-1}\text{Mpc}$ by Bertschinger et al (1990) and we use their values at two linear scales, 40 and $60h^{-1}\text{Mpc}$, to determine from eq.(4) the values of $\nu(0)$ implied by the data for various values of $\Omega^{0.6}/b$. The results are shown in the Table 1 below for $\gamma = 0.7$.

Table 1.	$\sqrt{\nu(r)}$	$\Omega^{0.6}/b$	$r = 40h^{-1}\text{Mpc}$	$r = 60h^{-1}\text{Mpc}$
			388 km/sec	330 km/sec
	$\sqrt{\nu(0)}$	1	1250 km/sec	1300 km/sec
		0.5	708 km/sec	709 km/sec
		0.3	526 km/sec	500 km/sec

The second row shows the values for the data for $\sqrt{\nu(r)}$ at 40 and $60h^{-1}\text{Mpc}$ used in computing $\sqrt{\nu(0)}$ shown in Table 1 against the values of $\Omega^{0.6}/b$. One can see from the Table that the values of $\Omega^{0.6}/b$ preferred by the data on $\xi(r)$, $\nu(r)$ and $\nu(0)$ are:

$$(0.2 - 0.3) < \frac{\Omega^{0.6}}{b} < (0.4 - 0.5) \quad (5)$$

This analytical estimate is consistent with our earlier results (Kashlinsky 1992a) and the results presented by Brent Tully at this meeting, but disagrees with the POTENT determination of these parameters (Yahil 1993, these proceedings).

5 Normalising to COBE

In order to determine/constrain $P(k)$ one must use the large scale data discussed in the previous sections in conjunction with the COBE observations on the MBR correlation function $C(\theta)$. However, such determination can at present be made reliably only on scales below the curvature radius, $R_{curv} = cH_0^{-1}/\sqrt{1-\Omega}$. Since the smallest scales subtended by COBE ($\sim 10^\circ$) exceed the curvature radius for values of Ω even as high as $\Omega \simeq 0.2 - 0.3$ (and the quadrupole scale always exceeds R_{curv} in open Universe) this analysis can be done only for the case of flat Universe. We therefore, concentrate in this section on discussing how well the standard inflationary scenario fits both the APM/peculiar velocity data which restrict $P(k)$ on scales $< 100h^{-1}\text{Mpc}$ and the COBE data which subtend scales $> 600h^{-1}\text{Mpc}$ if $\Omega=1$. In its conventional form, the inflationary scenario makes two predictions: 1) the Universe must be flat

($\Omega=1$) to a very high accuracy, and 2) the initial spectrum of the primordial density fluctuations must have the Harrison-Zeldovich form, $P(k) \propto k$. In the standard model fluctuations do not grow on sub-horizon scales during the radiation dominated era; on larger scales the growth would be self-similar. This leads to a unique shape of the transfer function accounting for the modification of the power spectrum, such that the Harrison-Zeldovich shape must be preserved on sufficiently large scales. (Constraints on the transfer function or $P(k)$ on small scales are discussed in the next section).

The first-year COBE data give the values of $\sqrt{C_{10^\circ}}$, the signal convolved with the 10° FWHM beam and the quadrupole anisotropy Q . For $P(k)$ given by (2) with the Harrison-Zeldovich spectrum ($n = 1$) and normalised to the APM data via k_0 we obtain within the uncertainty of the bias factor:

$$\sqrt{C_{10^\circ}(0)} \simeq \frac{2.6 \times 10^{-5}}{b} \frac{k_0^{-1}}{50h^{-1}\text{Mpc}} ; \quad Q \simeq \frac{1.2 \times 10^{-5}}{b} \frac{k_0^{-1}}{50h^{-1}\text{Mpc}} \quad (6)$$

One can see that it is possible to fit the COBE results if the bias factor is sufficiently large, $b > 2$. Indeed, the intrinsic uncertainty in the APM data would probably restrict k_0 to lie in the range $40h^{-1}\text{Mpc} < k_0^{-1} < 60h^{-1}\text{Mpc}$ with $k_0^{-1} = 50h^{-1}\text{Mpc}$ being the best fit. COBE data give $\sqrt{C_{10^\circ}(0)} = (1.1 \pm 0.18) \times 10^{-5}$ and $Q = (4.8 \pm 1.5) \times 10^{-6}$ (Smoot et al 1992), thus leading to $b = (2.3 \pm 0.4)$. One can tighten these constraints further by normalising the APM data to the observed peculiar velocities in the Great Attractor region, thus explicitly eliminating b (Kashlinsky 1992c). We do this using the peculiar velocity data at $r = 40h^{-1}\text{Mpc}$ from Bertschinger et al (1990). The numbers for the quadrupole anisotropy, Q , and the smoothed MBR correlation amplitude, $\sqrt{C_{10^\circ}(0)}$, we obtain are consistent within the error bars with the COBE results and can then be interpreted as supporting the standard inflationary picture.

We emphasize at the same time that inflation would be inconsistent with the COBE results and the large-scale structure data if either the transition to the HZ regime is less sharp than assumed in (2) or if more power is found on scales that currently cannot be probed by galaxy samples. Furthermore, the presence of the gravitational wave background, which is an inevitable consequence of inflation as discussed by Paul Steinhardt in these proceedings, would produce an extra contribution to (6) and lead to (much) larger values of b required by matching the APM data to COBE.

6 High- z objects: constraints on $P(k)$ on small scales

On scales which are non-linear today, $r < 8h^{-1}\text{Mpc}$, the APM data and eq.(2) give little direct information on the primordial form of $P(k)$. However, inflation makes also very robust predictions what this form should be once $P(k)$ is normalised to the large-scale data. Precisely because inflationary models have no free parameters (b can now be fixed as discussed above), the early evolution of density fluctuations would lead to a unique (for a given Ω_{CDM} , Ω_{HDM} and λ) transfer function thus also constraining the small scale power spectrum which is responsible for collapse of objects at high z . This was discussed by Cavaliere and Szalay (1986) and Efstathiou and Rees (1978) and in the context of the inflationary models normalised to the large-scale data by Kashlinsky (1993). We briefly review the results here.

As discussed above in order to account for all the data, inflationary models have to be normalised to the power spectrum seen in the APM catalog. I.e. the zero crossing of the two-point correlation function should occur on scale $r \simeq 2.5k_0^{-1} \simeq 100-150h^{-1}\text{Mpc}$ instead of $30(\Omega h)^{-1}h^{-1}\text{Mpc}$ for the standard, $\Omega=1$ and $h=\frac{1}{2}$, CDM model. Two ways have been suggested to overcome this problem and to increase the power on large scales: 1) Introduce the cosmological constant $\Lambda \equiv 3H_0^2\lambda$ such that $\Omega+\lambda=1$; in Efstathiou et al (1988) it is shown that such model with $\Omega h \sim 0.1-0.2$ would produce the large scale power seen in the APM. 2) Introduce two types of dark matter: HDM+CDM; e.g. Davis et al (1992) and Taylor and Rowan-Robinson (1992) shows that if $\Omega_{HDM} \simeq 0.3$ with the remaining contribution to $\Omega_{total}=1$ coming from CDM, such model gives good fits to a variety of large-scale structure data.

However, CDM/inflationary models would at the same time suppress the small scale power and hence have difficulty in accounting for the observed objects at $z > 3-4$. Indeed, for the $\Omega+\lambda=1$ models the transfer function is given by $T(k) = \{1 + [ak + (bk)^{3/2} + (ck)^2]^\nu\}^{-1/\nu}$, where $\nu=1.13$ and $a=6.4(\Omega h)^{-1}h^{-1}\text{Mpc}$; $b=3.2(\Omega h)^{-1}h^{-1}\text{Mpc}$; and $c=1.7(\Omega h)^{-1}h^{-1}\text{Mpc}$ (Bond and Efstathiou 1984). The range of $T(k) \simeq 1$ or the effective power spectrum index $n=1$ corresponds to scales where the Harrison-Zeldovich form of the power spectrum got preserved and so $P(k)$ enters the Harrison-Zeldovich regime for scales $> 2a$. On smaller scales the power index varies from $n \simeq 1$ through $n \simeq -1$, required by the APM data, to $n \simeq -3$ for scales $\ll c$. The scales where $n \simeq -3$ correspond to very little small scale power and this suppresses collapse of fluctuations (and galaxy formation) until a fairly low z for CDM models. Lowering Ωh increases a and thus can provide the power found in the APM survey; at the same time this increases $c \propto (\Omega h)^{-1}$ and further suppresses early collapse of density fluctuations on the relevant scales. A similar effect would be achieved if part of the contribution to Ω_{total} is due to HDM (van Dalen and Schaeffer 1992).

The predictions of so normalised CDM/inflationary models can and must be compared to the observational data on 1) QSOs at $z \geq 4.5$; 2) the recently found protogalaxies at $z \sim 4$; and 3) the protocluster-size object recently discovered by Uson et al (UBC) (1991). As discussed (Kashlinsky 1993) one can avoid the difficulty with the currently observed QSO abundances ($z \simeq 4.5$) mainly because of the freedom one has in determining their total collapsed masses (cf. Nusser and Silk 1993). But the data on the high- z galaxies ($z \simeq 4$ with the total collapsed masses of $> 3 \times 10^{12} M_\odot$ as the data indicate, see e.g. Chambers and Charlot 1990 and the references cited therein) would be difficult to account for on the basis on the modified CDM models. In other words, it may be difficult within the framework of CDM models to account simultaneously for 1) large-scale optical data; 2) COBE results and 3) high- z objects. I.e. if one normalises CDM models to the large-scale and COBE data, by lowering Ωh and putting in $\lambda=1-\Omega$ or by having HDM as well as CDM, thereby reducing the small-scale power of the density field, one should then expect to see 1) significant reduction in the QSO number densities at $z > (4-6)$ in any modified CDM models; 2) no protogalaxies collapse at $z \geq 4$; and 3) no protocluster-size objects, such as the UBC object, at $z \simeq 3.4$. Thus the data on the high- z galaxies may require a power spectrum that has more power on small scales than CDM models, e.g. one that has $n \sim -1$ on scales down to at least $10^8 M_\odot$, which scale-invariant inflationary models cannot provide. The existence of the UBC object would put even stronger constraints on hierarchical models: indeed,

this object corresponds to the comoving scale $\sim 8h^{-1}\text{Mpc}$; this fixes its r.m.s. density contrast to be b^{-1} at the present epoch almost independent of $P(k)$. Its existence at $z=3.4$ would in e.g. CDM models correspond to a $>10\text{-}\sigma$ fluctuation and one should not see any of such objects within the horizon. To reduce this number can be achieved by both 1) requiring the light to trace mass, i.e. $b=1$, and 2) making the Universe open, which would slow down collapse of fluctuations out to and lead to structures forming by $z_{in} \simeq \Omega^{-1} - 1$ as opposed to $z_{in} \simeq \Omega^{-\frac{1}{3}} - 1$ for $\Omega + \lambda = 1$ models. Such models would require to reconsider the validity of the standard inflationary assumptions.

7 Conclusions

In this review we have discussed the shape of the primordial power spectrum and the values of the cosmological parameters implied by the data. We have shown that the peculiar velocity field is in good agreement with that predicted by galaxy correlation data and that this suggests that the bias factor is constant at least in the linear regime. The comparison also allows one to estimate $\Omega^{0.6}/b$ and the data is consistent with $0.2 < \Omega^{0.6}/b \leq 0.5$. If the Universe turns out open, it would be impossible to fine-tune inflation as in that case the super-horizon scale inhomogeneities would induce, via the Grischuk-Zeldovich effect, MBR quadrupole in excess of the COBE data. We further discussed how 1) the microwave background data from COBE, 2) the optical data on the distribution of galaxies at $z \sim 0$, and 3) the data on high- z galaxies constrain the primordial $P(k)$ over a range of scales from $< 1h^{-1}\text{Mpc}$ to $> 1000h^{-1}\text{Mpc}$ and the implications of all the data for inflationary scenario(s). Bahcall, N. and Soneira, R.

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