Determining Ω from cluster correlation function

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Abstract. It is shown how data on the cluster correlation function can be used in order to reconstruct the density of the pregalactic density field on the cluster mass scale. The method is applied to the data on the cluster correlation amplitude – richness dependence. The spectrum of the recovered density field has the same shape as the density field derived from data on the galaxy correlation function which is measured as function of linear scales. Matching the two amplitudes relates the mass to the comoving scale it contains and thereby leads to a direct determination of Ω. The resultant density parameter turns out to be Ω=0.25.

1. Introduction

This paper presents another way of determining Ω by comparing density fields determined from two independent datasets: APM data on the galaxy two-point correlation function and the data on the cluster correlation function – richness dependence. For reasons that will become clear later in the paper, I will reconstruct the quantity which is uniquely related to the correlation function, $\xi(r)$, of the density field, or its Fourier transform - the power spectrum $P(k)$:

$$\Delta^2 = \langle(\delta M/M)^2\rangle = 4\pi\int_0^r \xi(r')r'^2dr' = 4\pi\int_0^\infty P(k)\left(\frac{3j_1(kr)}{kr}\right)^2 k^2dk \qquad (1)$$

I will limit the discussion and the formalism to scales where the present density field is linear and therefore can be assumed to reflect the initial conditions, i.e. $r > r_8 \equiv 8h^{-1}$Mpc.

The outline and the idea of the paper are as follows: in Sec.2 I discuss reconstruction of (1) as function of the linear scale, r, from the APM data. In Sec.3 I show how to use the data on the cluster correlation amplitude – richness dependence in order to reconstruct (1) as function of the cluster mass. Comparison of the two density fields gives the amount of mass contained in the given linear scale and provides a direct determination of Ω. I will show that the two density fields, although recovered in different and independent ways, give consistent results and require $\Omega \simeq 0.25$ with a very small uncertainty. For more details the readers are referred to [1].

2. Density field from galaxy clustering

First, I establish the rms fluctuation, $\Delta(r)$, as function of the linear scale r from the APM data on the projected angular correlation function $w(\theta)$. For this I use the APM data on $w(\theta)$ [2] divided into six narrow magnitude bins $\Delta m_b \simeq 0.5$. Galaxies located in each of the bins span a narrow(er) range of z, so with the data presented in this way one can isolate effects of possible galaxy evolution in the entire APM catalog spanning $17<b_J<20$ or $0.07<z<0.2$ [3].

The projected angular correlation function for each bin is related to the 3-dimensional power spectrum of galaxy clustering, $P(k)$, via the Limber equation:

$$w(\theta) = \pi \int_0^\infty dz(cdt/dz)^{-1}\phi(z) \int_0^\infty dk P(k;z) k J_0(\frac{kx(z)\theta}{1+z}) \tag{2}$$

where $x(z)$ is the comoving distance, t is the cosmic time, and the selection function is $\phi(z) = (dN/dz/N_{\rm tot})^2$. The latter is related to the range of magnitudes for galaxies in each bin, $[m_l, m_u]$, and their luminosity function, $\Phi(L)$, via $dN/dz = (dV/dz) \int_{L(m_u)}^{L(m_l)} \Phi(L;z)dL$ with V being the comoving volume. Eq.(2) allows to relate the data on $w(\theta)$ to the underlying power spectrum for each of the six narrow magnitude width bins once all of the following are specified: the luminosity function in a given band (blue for APM), the K-correction which accounts for the shift along the galactic energy spectrum resulting from cosmic expansion, and the (possible) galaxy evolution out to the edge of the APM sample. For the numbers in the remainder of this section the luminosity function was adopted from the measurements of [4]; the K-correction was modeled from standard spectra of galaxy populations [5].

In order to estimate the importance of the (possible) evolution effects, I proceeded as follows: On very small angular scales, the data show that the angular correlation function can be described as a power law, $w(\theta) = A_w\theta^{-\gamma}$ with $\gamma = 0.7$. On the other hand, as $\theta \to 0$, the main contribution in the Limber equation comes from very small linear scales where the spatial galaxy correlation function can be approximated as $\xi(r)=(r/r_*)^{-1-\gamma}$ with $r_* = 5.5h^{-1}$Mpc. Relating the small-scale $\xi(r)$ to $w(\theta)$ via the Limber equation leads to the following expression for A_w:

$$A_w = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{\gamma}{2})}{\Gamma(\frac{1+\gamma}{2})} \left(\frac{r_*}{cH_0^{-1}}\right)^{1+\gamma} \int_0^\infty \phi(z) \left[\frac{cH_0^{-1}(1+z)}{x(z)}\right]^\gamma \Psi^2(z)(1+z)^2\sqrt{1+\Omega z}dz \tag{3}$$

where $\Psi(z)$ accounts for the evolution galaxy clustering, e.g. $\Psi^2(z) \propto (1+z)^{-3}$ for clustering stable in comoving coordinates. Comparing the value of $A_w = w(\theta)\theta^{0.7}$ computed from eq.(3) with the data for each bin allows to constrain the extent to which galaxy evolution affects inversion of the APM data in terms of the underlying spectrum. The fits of the power-law galaxy correlation function to the small-scale data on $w(\theta)$ in all six narrow magnitude bins show that no galaxy evolution corrections are needed beyond the normal K-corrections and evolution of the clustering pattern with time [1].

Analysis of the data in narrow magnitude bins without galaxy evolution shows that the power spectrum obtained by deprojection of the entire APM dataset [6] fits the data well in all magnitude bins. Neglecting the dependence on $\Omega_{\rm baryon}$, CDM power spectra can be parameterized by only two parameters: the primordial power index n and the excess power parameter Ωh [7]. Fits of the CDM models to APM data in narrow magnitude bins at all depths show that the models require Ωh=0.2 if n=1, or Ωh=0.3 for tilted models with n=0.7. The latter thus still requires low Ω in order to fit the APM data in narrow magnitude bins. *CDM models with larger values of Ωh would give smaller $w(\theta)$ at large θ; whereas smaller values would overshoot the data* [1].

Left panel in Fig.1 shows the rms fluctuation from the fits to the APM data vs the comoving scale. It is plotted in units of the fluctuation at $r_8 \equiv 8h^{-1}$Mpc, $\Delta(r)/\Delta_8$. The reason for plotting the ratio is that for linear biasing the vertical axis in Fig.1 is independent of the bias factor.

3. Density field from cluster correlation amplitude – richness dependence

In this section I show how, using the data on the cluster correlation amplitude – richness dependence, one can reconstruct the quantity plotted in the left panel of Fig.1 as function of the cluster mass. Comparing the results with Δ over a given range of r gives the amount of mass in the given comoving scale and leads to a direct determination of Ω.

I assume that clusters of galaxies formed by gravitational clustering [8] and evaluate their correlation function based solely on this assumption [9, 10]. I.e., I assume that clusters of galaxies are identified with regions that at some early epoch z_i had initial overdensity such that they would turn-around in less than the age of the Universe. It is convenient to choose z_i to be sufficiently high when the density field on all relevant scales is in the linear regime. In that case, the amplitude, δ_{ta}, of the fluctuation at z_i needed for it to turn around today at z=0 is related to $\Delta_{8,i}$, the amplitude which grows to Δ_8=1 at z=0, via $\delta_{ta} = Q_{ta}\Delta_{8,i}$. The factor $Q_{ta} \simeq 1.65$ for a spherical model and is almost independent of either z_i or cosmological parameters.

I further assume that the initial density field was Gaussian. In that case the joint probability density to find density contrasts $\delta_{1,2}$ on scales containing masses $M_{1,2}$ respectively is given by:

$$p(\delta_1;\delta_2) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp(-i\boldsymbol{q}\cdot\boldsymbol{\delta})\exp(-\frac{1}{2}\boldsymbol{q}\cdot\boldsymbol{C}\cdot\boldsymbol{q})d^2q \tag{4}$$

The correlation matrix, $\boldsymbol{C}$, is related to the spectrum of the *primordial* density field: its diagonal elements are the mean square fluctuation Δ^2 on scale containing mass M and the non-diagonal elements are $\simeq\xi(r)$ at cluster separations r greater than the comoving scale subtended by the cluster masses ($\sim$1-3 h^{-1}Mpc). The probability of two such fluctuations to turn-around by now and thereby form clusters of galaxies is $P_{M_1M_2} = \int_{\delta_{ta}}^{\infty}\int_{\delta_{ta}}^{\infty} p(\delta_1;\delta_2)d\delta_1 d\delta_2$. The fraction of such pairs at the present time would be $f_{M_1M_2} = \partial^2 P_{M_1M_2}/\partial M_1/\partial M_2$. The probability for a single cluster to form by now is $P_M = \int_{\delta_{ta}}^{\infty} p(\delta)d\delta$; the fraction of such clusters is $f_M = \partial P_M/\partial M$.

Now one can construct the correlation function between the present-day clusters of different masses. By definition, the 2-point correlation function of an ensemble of objects with number density n is given by the probability to find two objects in small volumes dV_1, dV_2 as $d\mathcal{P}_{12} = n^2(1+\xi)dV_1dV_2$. Since the clusters of mass M_1, M_2 will make the fraction $f_{M_1M_2}$ of such pairs, the probability to find them is $d\mathcal{P}_{M_1M_2} = f_{M_1M_2}d\mathcal{P}_{12}$. On the other hand, the mean number density of clusters of mass M would be $f_M \times n$ and by definition the probability to find two clusters is $d\mathcal{P}_{M_1M_2} = f_{M_1}f_{M_2}n^2(1+\xi_{M_1M_2})dV_1dV_2$. Hence the correlation function of clusters of mass M is given by:

$$1+\xi_{M_1M_2} = \frac{f_{M_1M_2}}{f_{M_1}f_{M_2}}(1+\xi_i) \tag{5}$$

Using expansions in terms of Hermite polynomials [11, 10] leads to the following expression [1]:

$$\xi_{MM}(r) = A_M(\frac{\zeta}{\sqrt{2}})\xi(r) \tag{6}$$

where the amplification factor is:

$$A_M(x) = \sum_{m=0}^{\infty} \frac{1}{Q_{ta}^{2m} m!} \xi^m C_m(x) \tag{7}$$

with

$$C_m(x) = \frac{x^{2m}}{4} \left[\frac{H_{m+1}^2(x)}{x^2} + \frac{H_{m+2}^2(x)}{(m+1)Q_{ta}^2} \right] \tag{8}$$

Here $H_m(x)$ are Hermite polynomials and $\zeta{=}Q_{ta}\frac{\Delta_8}{\Delta(M)}$, which is directly related to the spectrum of the primordial density field on scale M. Hence, the data on the cluster correlation amplitude – mass dependence [12, 13] can be used to invert eqs.(5)-(7) to directly obtain $\Delta(M)$ for the *primordial* density field. The accuracy of these expressions has been confirmed in recent numerical experiments [15].

Since cluster correlation amplitude is known to depend on the cluster richness, $\mathcal{N}$, one has to translate the richness into cluster mass. I assume that the two are proportional with the coefficient of proportionality normalized to the data on the Coma cluster:

$$M(\mathcal{N}) = M_{\rm Coma} \frac{\mathcal{N}}{\mathcal{N}_{\rm Coma}} = 1.45 \times 10^{15} \left(\frac{\mathcal{N}}{106} \right) h^{-1} M_{\odot} \tag{9}$$

Here, I adopted the values for the Coma cluster and richness following [14]. This assumes that there are no systematic variations in the galaxy luminosity function in clusters of various masses.

The data on the cluster correlation function can now be used in conjunction with this mechanism for cluster formation in order to set further constraints on the cosmological models via eqs.(5)-(7). Indeed, the dependence of the cluster correlation function at a given richness/mass on r constrains the power spectrum on the scale of cluster separation, whereas the dependence of the cluster correlation amplitude at a given r on the cluster richness/mass constrains the shape of the *primordial* power spectrum on the scale containing that mass. Eqs.(5)-(7) can be used in two ways: On the one hand, one can test cosmological paradigms, such as CDM, by evaluating the cluster correlation parameters by assuming their power spectrum and comparing computed numbers for both $\xi_{MM}(r)$ vs r and A_M vs M with observational data. Alternatively, one can use these expressions in conjunction with data on the cluster correlation amplitude in order to directly obtain the spectrum of the *primordial* density field on scale M independently of the assumed cosmological prejudices. The latter, as I show, can also be used to determine Ω.

In terms of the CDM models, I find that no CDM model, whose only free parameters are (Ω, h, n), can simultaneously fit three sets of data: the APM data on the galaxy correlation, the data on the slope of the cluster correlation function with scale at a given richness, and the data on the dependence of the cluster correlation amplitude at a given separation on the cluster richness. Namely, the values of (Ω, h, n) required by fits to the APM data ($\Omega h \simeq 0.2$ if $n = 1$ and $\Omega h \simeq 0.3$ if $n = 0.7$) would be very different from those required by fits to either of the other two datasets and vice versa [1]. Accounting for a (slight) dependence on $\Omega_{\rm baryon}$ in the CDM transfer function [16] would not change these conclusions.

In order to invert eq.(6) in terms of x, and subsequently $\Delta(M)$, I used the data on the cluster correlation amplitude – richness dependence from Fig.2 of [13]. The amplification coefficient for a given richness/mass in eqs.(5)-(7) depends on the value of $\xi(r)$ on the scale where it is evaluated. The underlying correlation function $\xi(r)$ enters eq.(6) via the first and higher order terms and can contribute up to ~10-30% to the total amplification. I chose r=25h^{-1}Mpc on which to evaluate the amplification factor from (7). This scale is sufficiently large compared to r_8 to ensure validity of the analysis, but where at the same time $\xi(r)$ can be determined sufficiently accurately. In the discussion below I adopted $\xi(25h^{-1}\mathrm{Mpc})$=0.07 in agreement with the APM data (cf. [1]), but the numbers that follow are not very sensitive to varying the value of $\xi(25h^{-1}\mathrm{Mpc})$ within reasonable limits.

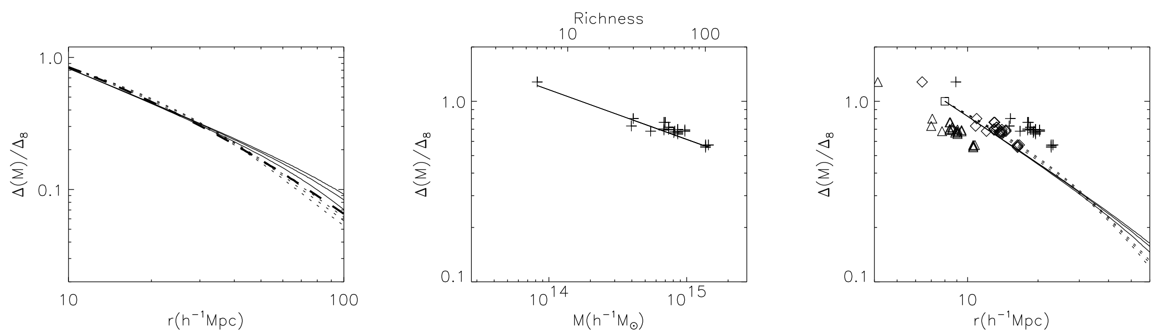

Figure 1. (Left) RMS density fluctuation from APM data is plotted vs the comoving scale. It is shown in units of density fluctuation over radius r_8. Solid lines correspond to empirical fit from [3]; dotted lines to deprojected spectra from [6]. Three lines of each type correspond to 1-σ uncertainty in [6] and a similar uncertainty in [3]. Thick dashed line corresponds to CDM models that fit APM data in narrow magnitude bins: Ωh=0.2 with n=1 or Ωh=0.3 with n=0.7. (Middle) Plus signs correspond to the primordial density field inverted from data on cluster correlation amplitude – richness dependence. Solid line shows the fit to the points: $\Delta(M)$=$\Delta_8(M/1.7\times 10^{14}h^{-1}M_\odot)^{-0.275}$.
(Right) The values of $\Delta(r)$ from the cluster data for three different values of Ω: plus signs correspond to Ω=0.1, triangles to Ω=1 and diamonds correspond to Ω=0.25. Square denotes the (by definition) value of unity of $\Delta(M)/\Delta_8$ at 8h^{-1}Mpc. The lines represent density fluctuations derived from the APM data redrawn from the left panel in the same notation.

Once the data on the rhs of eq.(7) for A_M vs richness are specified, eqs.(5)-(7) can be solved numerically in order to obtain $\Delta(M)$. The middle panel in Fig.1 plots the results of this inversion. The top horizontal axis plots the values of $\mathcal{N}$ at which $\Delta(M)/\Delta_8$ has been evaluated. The bottom horizontal axis shows the mass computed according to the normalization to Coma. I emphasize again that this method gives directly the pregalactic spectrum as it was at z_i independently of the later gravitational or other effects. The plot in the middle panel of Fig.1 shows a clearly defined slope of $\Delta(M)\propto M^{-0.275}$ corresponding to the spectral index of $n\simeq -1.3$. This slope is consistent with the APM implied power spectrum index of $w(\theta)\propto\theta^{-0.7}$.

4. Determining Ω from the two density fields

The recovered density field allows one to relate the mass of the cluster to its comoving scale thereby directly determining Ω. The direct fit to the points in the middle panel of Fig.1 gives

$$\Delta(M)=\Delta_8(M/M_8)^{-\alpha} \quad (10)$$

with α=0.275 and $M_8=1.7\times10^{14}h^{-1}M_\odot$. This fit is plotted with solid line in the middle panel. On the other hand, the mass contained in comoving radius r_8 is $M(r_8)=6.1\times10^{14}\Omega h^{-1}M_\odot$. Equating this with M_8 leads to Ω=0.28.

Furthermore, one can determine Ω by comparing $\Delta(r)$ from the galaxy correlation data over the entire range of the relevant r with $\Delta(M)$ derived from the cluster correlation amplitude – richness dependence. In order to do this I converted the numbers for $\Delta(M)$ to those at a given r using that the mass contained in a given comoving radius in the Universe with density parameter Ω is $M(r)=1.2\times10^{12}(r/1h^{-1}\text{Mpc})^3\Omega h^{-1}M_\odot$. The right panel in Fig.1 shows the values of $\Delta(r)$ from the cluster data for three different values of Ω: 0.1 (pluses), 0.25 (diamonds) and 1 (triangles). The square denotes the (by definition) value of unity of $\Delta(M)/\Delta_8$ at $8h^{-1}$Mpc. The lines represent the APM data redrawn from the left panel. One can see that the spectral shape of the density field derived from the cluster data is in good agreement with that of the APM. *The amplitudes of the two fields would match at all r only for Ω=0.25.*

5. Summary and conclusions

In this presentation I discussed reconstructing density field from the cluster correlation amplitude – richness dependence. It was shown that the data can be inverted to obtain the rms density fluctuation in the pregalactic density field on scales containing the mass of the clusters. The derived density field has the same spectral shape as the density field derived as function of the comoving scale from the APM data. Comparing the two amplitudes fixes the amount of mass in given comoving scale and allows to determine Ω. The value derived from application of the method to the data is Ω=0.25. This value of Ω, obtained after normalizing the mass-richness relation to Coma, is in good agreement with that implied by the dynamics of the same Coma cluster. This further argues that galaxies trace the overall mass distribution in the Universe.

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